

Retinal Blood Vessels Segmentation using Fréchet PDF and MSMO Method

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Abstract

Abstract: Blood vessels of retina contain information about many severe diseases like glaucoma, hypertension, obesity, diabetes etc. Health professionals use this information to detect and diagnose these diseases. Therefore, it is necessary to segment retinal blood vessels. Quality of retinal image directly affects the accuracy of segmentation. Therefore, quality of image must be as good as possible. Many researchers have proposed various methods to segment retinal blood vessels. Most of the researchers have focused only on segmentation process and paid less attention on pre processing of image even though pre processing plays vital role in segmentation. The proposed method introduces a novel method called multi-scale switching morphological (MSMO) for pre processing and Fréchet match filter for retinal vessel segmentation. We have experimentally tested and verified the proposed method on DRIVE, STARE and HRF data sets. Obtained outcome demonstrate that performance of the proposed method has improved substantially. The cause of improved performance is the better pre processing and segmentation methods.

Key Words: Image enhancement, MSMO Operator, Segmentation, Fréchet PDF.

1 Introduction

Illness in any form is not desirable and everyone wants to get rid of it. There are numbers of severe diseases which affect human life badly today. Some of them are hypertension, diabetes, glaucoma, obesity, macular degeneration, arthritis etc. Glaucoma and diabetes can cause vision loss too. Diabetic, obesity and hypertension are the bases for several deadliest diseases. Diseases are generally identified either by their symptoms or by color and structure. Diabetes, glaucoma, obesity, macular degeneration, hypertension etc. can be identified by observing color and structure of retinal vessels. Hence, segmentation of retinal's vessels is necessary. During the process of capturing and transmission of retinal images, several factors affect the quality of the images. Generally, retinal images are too dark, too bright, unclear and have poor contrast, noises, artificial colors, etc. Due to these, quality of retinal images is generally poor. Quality of a retinal image directly affects the accuracy of segmentation of vessels. Therefore, there is need an approach for better image enhancement as well as segmentation of retinal vessels [17][18].

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Image enhancement has always been challenging and main task of image processing. Characteristics of retinal images make it more tedious. In the mean, there is need an appropriate method for retinal image enhancement. There are existing various methods today for image enhancement. These are classified as in the Figure 1. Various approaches are given by many researchers for image enhancement, but they are not efficient and have scope of improvement.

In the proposed approach, we have applied the MSMO operator along with principal-component analysis (PCA) and contrast-limited adaptive histogram equalization (CLAHE) methods for pre processing and Fréchet match-filter for vessel segmentation. MSMO operator is a morphological operator. It is used to enhance the retinal images. It is more efficient and gives better results for not only retinal images but also for MRI, ultrasound, X-ray images etc. too. Fréchet match filter is a latest and one of the best segmentation methods for retinal vessels. Fréchet match filter performs better than others when it is applied on enhanced grayscale image by the MSMO operator. The main objective of this literature is to maximize the accuracy of segmentation. The proposed method is more efficient as well as robust. The major contributions of the literature are:

- Detailed study of various image enhancement techniques.
- Design of a methodology based on MSMO operator for retinal or medical image enhancement.
- Design of a novel approach based on MSMO operator and Fréchet PDF for better segmentation.
- Conducted in-depth experiments on large numbers of images to examine and verify the method.

Remaining part of the literature is arranged as: In Section-2, various related findings are investigated for better image enhancement and vessel's segmentation. In Section-3, a novel approach is presented for better image enhancement and vessel's segmentation. In Section-4, performance of the method is analyzed. At last, the method is concluded in Section-5.

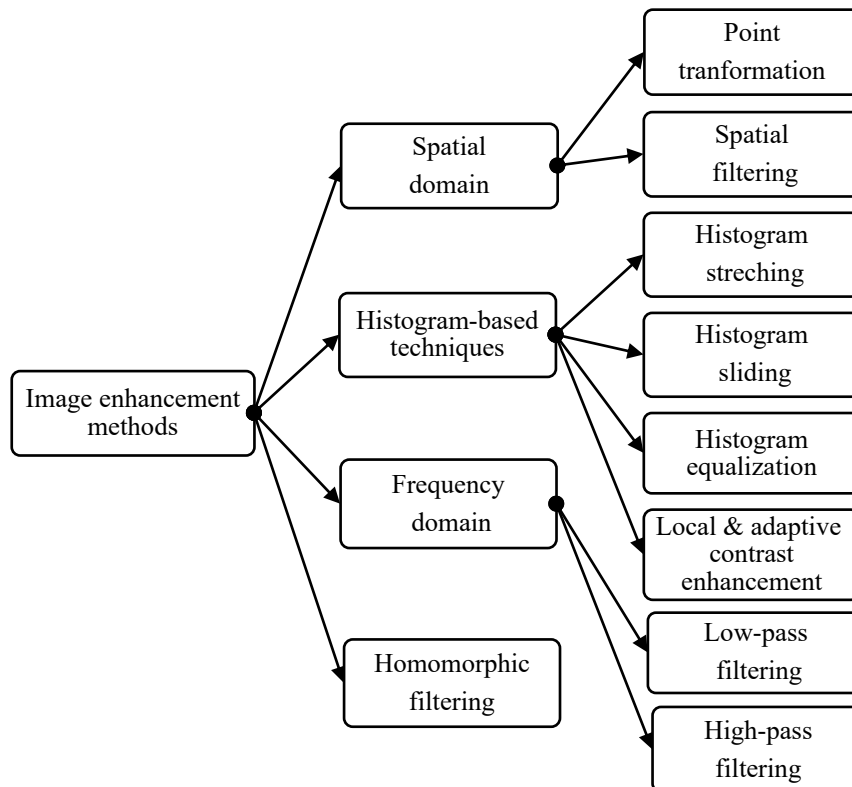


Figure 1: Taxonomy of image enhancement techniques

2 Related Work

The Sobel, Canny, Gradient, and Prewitt operators are the classical methods for edge detection. They work well when edges are distinct as well as sharp. Although, retinal blood vessels are generally thin and entangled. Intensities of vessels change gradually and contrast between vessels and their background are generally very low. In these scenarios, these classical methods fail to detect retinal vessels [23][24][25][29].

Accuracy of vessels segmentation produced by various methods is generally low. The main reason behind this is retinal vessel has very low contrast corresponding to its background. Contrast between vessel and its background can be increased by applying an efficient image enhancement method. Therefore, there is required an approach that not only good in vessel segmentation but also in pre processing. In search of this, various prominent methods based on pre processing and segmentation of retinal vessels are investigated. Some of them are discussed below [2][10]:

Singh et al. [7] have given an approach for vessels segmentation. In the approach, they have used the PCA and CLAHE methods to improve the quality of retinal images. They have applied the Gumbel match filter on obtained enhanced retinal images to generate the match-filter response (MFR) images. Author has applied the optimal thresholding technique to extract the retinal vessels from MFR images. They said that vessel profile shape does not oriented uniformly towards its trunk-value but little bit skewed. In contrast, Gumbel PDF is also little bit skewed about its trunk-point. Hence, author has implemented the Gumbel PDF as the kernel for their match filter. Even though, vessel- profile shape is not really Gumbel-curve shape. It is Fréchet curve shape. Image enhanced by above methods is not so good. Also, accuracy of segmentation is low.

Kaba et al. [8] have introduced a method for retinal vessels segmentation. In their approach, they have used the adaptive histogram equalization and bias correction in pre processing stage to enhance the retinal images. They have applied the probabilistic modelling on obtained enhanced images to extract the vessels. Expectation maximization technique is used for optimization. This model is effective only in tracking-based scenario. The model reduces width of original vessel. Enhanced images using the adaptive histogram equalization and bias correction is not good enough. Due to which accuracy of segmentation is poor.

Odstrcilik et al. [4] have designed a method for vessel's segmentation. In the approach, they have used contrast equalization and illumination correction techniques in the pre processing stage for image enhancement. They have used green channel of RGB image. Based on widths of vessels, intensity profiles of vessels are divided into five categories here. The author has designed five two-dimensional match filter kernels for each image. Due to five match filter kernels for each image, computational complexity increased five times. Also, accuracy of the model relies on how accurately the author divides intensity profile into five parts.

Zhou et al. [5] have given an approach for vessels segmentation. In the approach, author has applied the 2 D Gabor wavelet for image smoothing and a mathematical operation for contrast normalization and luminosity equalization in the pre processing stage. They have applied convolutional neural network (CNN) and dense CRF method for segmentation of vessels. Image enhanced by above methods is good but not sufficient due to which accuracy is comparatively low.

GeethaRamani et al. [21] have proposed an algorithm for vessels segmentation. In their literature, they have used the Gabor filter and Halfwave rectification methods in pre processing stage to crop image, transform color and extraction color channel. From preprocessing stage, we obtain a feature vector. The PCA method is applied on this feature vector. Thereafter, k means clustering technique is applied on obtained result to categories pixels in the vessels and non-vessels groups. Ensemble classifier removes the non vessel group. The model is quite complex. Image enhanced in preprocessing stage is not good enough due to which accuracy of segmentation is low.

Memari et al. [1] have introduced a method for vessel-segmentation. In this literature, the author has applied the CLAHE and mathematical morphology methods for retinal image enhancement. The author has used the Frangi and Gabor match filters to enhance the vessels networks. The Improved spatial fuzzy c means using genetic algorithm is applied for vessels segmentation. The model has high computational overhead. Retinal images enhanced by this method is not so good. Also, accuracy of segmentation of the model is relatively low.

Saroj et al. [6] have proposed a method for vessel's segmentation. In their literature, they have applied the PCA and CLAHE methods for retinal image enhancement. They have applied the Fréchet match filter for segmentation of vessels. They have tested their approach on STARE as well as DRIVE data sets. Although, segmentation method is efficient and good but methods used for image enhancement seems inappropriate and less efficient. Due to which, method did not perform as it could.

The methodology proposed in the literature involves a novel method for image enhancement and an efficient match filter for vessels segmentation. Blood vessels of retina have usually very low contrast with their background due to which vessels are not fully extracted from their backgrounds. MSMO is a novel and efficient method which enhances contrast between vessels and their backgrounds. Match filter method increases the contrast of retinal image. Hence, match-filter method identifies retinal blood vessels more correctly than other segmentation methods. Since Fréchet match filter is a latest and efficient match filter, we use this for vessel segmentation in the literature. Fréchet match filter along with MSMO method produced higher accuracy.

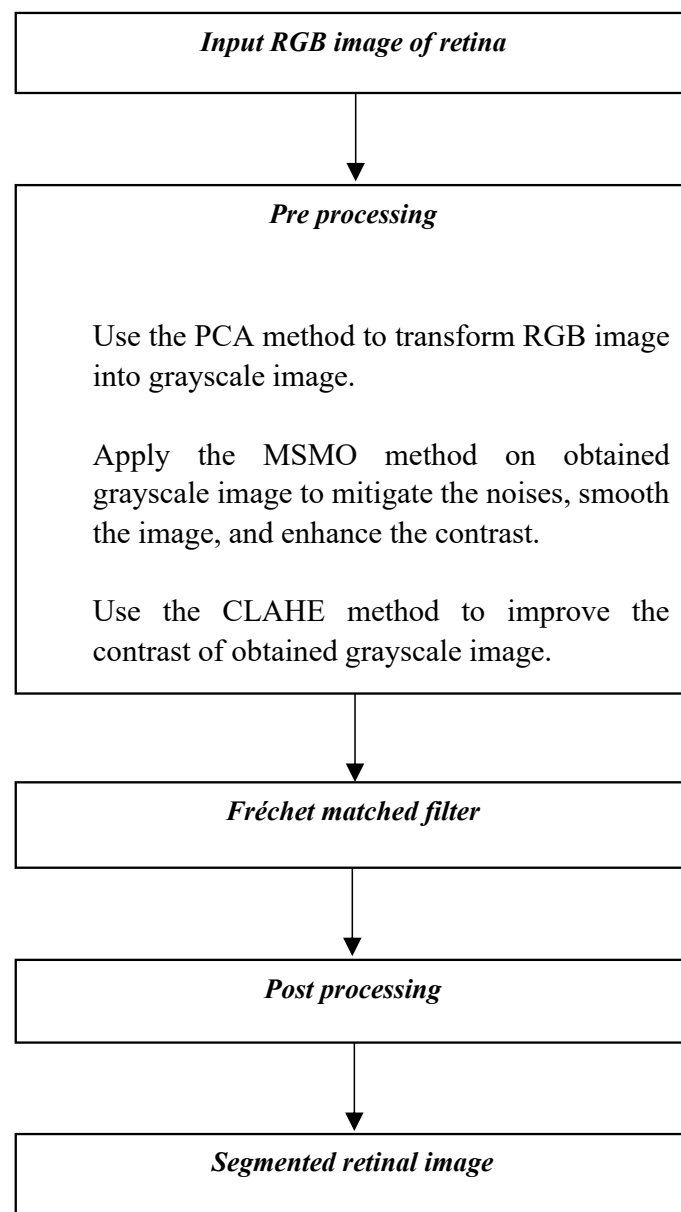


Figure 2: Block diagram of the proposed method

3 Proposed Method

The proposed method is based on a novel image enhancement technique i.e. the MSMO method and a best-known match filter i.e. Fréchet match filter for vessel segmentation. There are involved numbers of image operations in the approach which are grouped in the three major phases viz; pre processing, Fréchet match filter and post processing. Figure 2 shows these image-processing phases.

3.1 Pre-processing

Retinal vessels have few specific properties. Retinal vessels become thinner as we proceed out from optic disk. Intensity of retinal vessels change smoothly. Also, contrast between retinal vessel and its background is usually very low. Due to these characteristics, it is very hard to detect retinal blood vessels. Hence, to increase accuracy of vessel's segmentation, there is required to enhance the quality of retinal images. To achieve this, pre-processing of retinal image is needed. Generally, researchers emphasize more on only segmentation methods and pay less attention on pre processing although pre processing plays vital role in vessels segmentation. Therefore, we introduced a novel and efficient method i.e. the MSMO method for image enhancement. In the literature, pre processing module has three steps which are described as follows:

3.1.1 PCA Method

As we know that almost all the methods work on grayscale images. Therefore, it become necessary to convert RGB image into grayscale image. Most of the authors have used the `rgb2gray` method of MATLAB for this purpose. One more method called PCA is also exploited for this purpose. PCA method [12] has several advantages over `rgb2gray` method such as it preserves texture as well as color properties discriminability effectively. Also, it does not require any user specific parameter for conversion. Therefore, we use PCA for RGB to grayscale image conversion in the literature. But alone PCA method does not do much for image enhancement.

3.1.2 MSMO Method

We know that morphological operations such as dilation or erosion works on two inputs i.e. the grayscale image and structure element. Dilation (\oplus) as well as erosion (\ominus) operations on gray-scale image 'A' using structure element 'B' are defined by equation (1) and (2).

$$D = A \oplus B = \{(x, y) + (u, v) : x, y \in A; u, v \in B\}; \quad (1)$$

$$E = A \ominus B = \{(x, y) - (u, v) : x, y \in A; u, v \in B\}; \quad (2)$$

SMO is a morphological operator that uses the dilation and erosion operations as its sub functions. The operation dilation or erosion increases or decreases the tiny regions of an image according to size of structure element. Then matching with actual image, outcome of erosion or dilation operation primarily modifies tiny regions of the image. Due to this, SMO smooths regions of the image and replaces grey values of tiny regions with grey values of dilation or erosion operation. SMO is defined as follows:

```

if  $((D - A) < (A - E))$  then
    SMO=D;
else
    if  $((D - A) > (A - E))$  then
        SMO=E;
    else
        SMO=A;

```

end if
end if

Above definition of SMO implies that every pixel of enhanced image by SMO is particularly substituted by the same pixel in dilation, erosion results with a grey-value which is closed to grey-value of same pixel in the actual image. However, SMO with small size structure element is less efficient whereas SMO with large size structure element yields more noise. To reduce noise and make SMO more efficient, it is applied at multiple stages with structure elements having different sizes. Since SMO is applied on multiple scales with varying size of structure elements, it is called multi-scale SMO (MSMO). Some most frequently applied shapes into the structure elements are circle, square, hexagon and rectangle. Hexagon, rectangle and square shapes of structure elements generally have rough edges and yield block-effect which crash image region shape, particularly when structure element size is large. Circle shape of structure element has smooth edge due to which block-effect is very low. Hence, we opt circle shape as a shape of structure element in our literature. Scale number (n) is essential factor in the MSMO. Large n generally leads to heavy noise whereas small n is inefficient for retinal or medical image enhancement. The MSMO algorithm is described as follows:

Algorithm:

- Input gray scale image ‘A’ and structure element ‘B’.
- Perform dilation as well as erosion operations on ‘A’ using ‘B’ with size s for every scale s ($1 \leq s \leq n$).
- Outcome of SMO is computed for every scale s ($1 \leq s \leq n$).
- End result is formed along pixel wise taking the mean of all multi-scale outputs generated by SMO.
- Get enhanced gray-scale image.

MSMO method is operated on gray scale image generated by PCA method. It removes noises. It smooths images. It also enhances the contrast. Due to these improvements, accuracy of vessel segmentation has increased substantially.

3.1.3 CLAHE Method

CLAHE method is designed to improve the contrast of an image. It can work with homogeneous as well as heterogeneous images. It can work with gray scale as well as color images too. Gray-scale image generated by CLAHE method has less noise. Although, there are a lot of advantages of the CLAHE method but alone CLAHE or together with PCA is not sufficient in many cases. In the literature, CLAHE is applied on the output of MSMO method. PCA, MSMO and CLAHE methods combination produce better quality of image [22].

Signal to noise ratio (SNR): It is a metric which is used to compare the quality of images.

$$SNR = \mu/\delta \tag{3}$$

where, μ is the mean of an image, and δ is the standard deviation of an image.

Figure 3 shows the grayscale images generated or enhanced by different pre processing methods. From the Figure 3, it is clear that image enhanced by PCA+MSMO+CLAHE method is better than the images enhanced by rgb2gray, PCA and PCA+CLAHE. If we observe the Tables 1-5 then we find that SNR values obtained by PCA+MSMO+CLAHE for all the images of all data sets mentioned above are greater than SNR values obtained by rgb2gray, PCA and PCA+CLAHE. Value of SNR of the image is determined by equation (3). Table 6 shows the accuracy of segmentation for all images of DRIVE data set. These accuracies are produced by the Fréchet match filter when it is operated on the enhanced grayscale images by different pre processing methods. From the Table 6, it is clear that accuracy of segmentation of enhanced gray scale image by the PCA+MSMO+CLAHE is higher than rgb2gray, PCA and PCA+CLAHE. Alone PCA and CLAHE or PCA+CLAHE is not sufficient for retinal image enhancement. When we use the MSMO method with PCA and CLAHE then this combination yields better results as we can see in Tables 1-6 and Figure 3. Due to these reasons, we have applied a

novel method called the MSMO method with PCA and CLAHE in PCA+MSMO+CLAHE sequence for image enhancement. The MSMO along with PCA and CLAHE methods not only produce better results for retinal images but also for other medical images such as MRI, Ultrasound X-ray etc. as we can see the results in the Tables 1-5. Figure 4 shows the segmented images generated by proposed method using different pre-processing methods where PCA+MSMO+CLAHE has segmented image better. In the Tables 1-6, R, P, M, and C stand for rgb2gray, PCA, MSMO, and CLAHE, respectively.

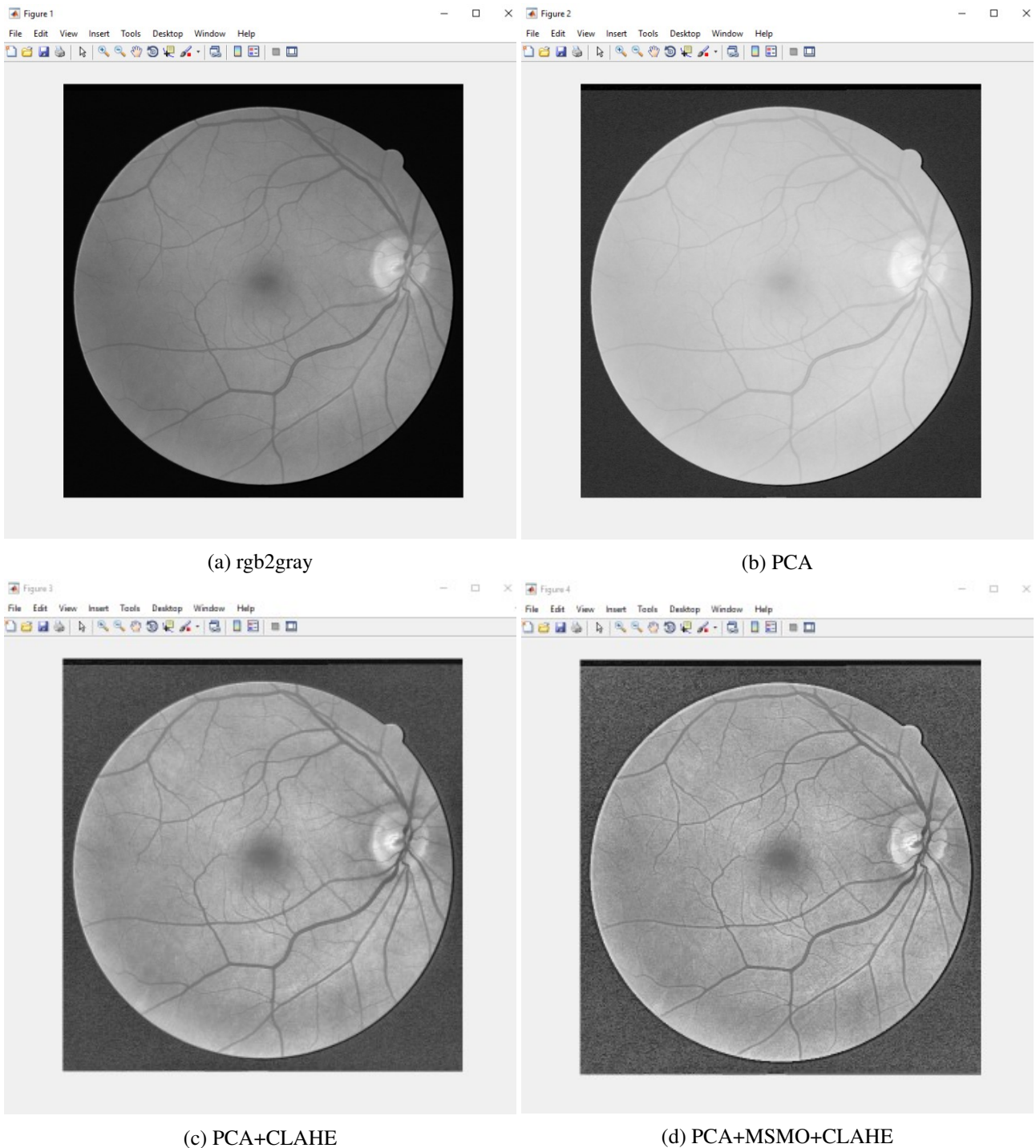


Figure 3: Enhanced retina images of 16_test.tif using different preprocessing methods

| Img | R | P | P+C | P+M+C |
|----------|---------|---------|---------|---------|
| 1 | 1.5654 | 2.1813 | 3.1036 | 3.2095 |
| 2 | 1.5492 | 2.1407 | 3.0182 | 3.158 |
| 3 | 1.6276 | 2.3632 | 3.074 | 3.2388 |
| 4 | 1.4961 | 2.1583 | 3.0286 | 3.1563 |
| 5 | 1.5778 | 2.219 | 3.1367 | 3.2668 |
| 6 | 1.575 | 2.1844 | 3.0115 | 3.1019 |
| 7 | 1.5115 | 2.1581 | 2.9985 | 3.1212 |
| 8 | 1.5394 | 2.1793 | 3.0693 | 3.2084 |
| 9 | 1.5561 | 2.1441 | 3.087 | 3.2088 |
| 10 | 1.5563 | 2.2397 | 3.1987 | 3.2591 |
| 11 | 1.5127 | 2.1405 | 3.0198 | 3.1569 |
| 12 | 1.5665 | 2.1806 | 3.1186 | 3.1883 |
| 13 | 1.5409 | 2.1424 | 3.0469 | 3.1804 |
| 14 | 1.5757 | 2.2267 | 3.1678 | 3.2818 |
| 15 | 1.4596 | 2.2231 | 3.1943 | 3.2741 |
| 16 | 1.561 | 2.1467 | 3.0021 | 3.3339 |
| 17 | 1.5635 | 2.1298 | 2.9637 | 3.091 |
| 18 | 1.5528 | 2.134 | 3.0329 | 3.1721 |
| 19 | 1.581 | 2.263 | 3.0688 | 3.1272 |
| 20 | 1.5713 | 2.1945 | 3.1684 | 3.3093 |
| Avg. SNR | 1.55197 | 2.18747 | 3.07547 | 3.20219 |

Table 1: SNR values for DRIVE images generated by different pre processing methods

| Img | P | P+C | P+M+C |
|----------|---------|----------|---------|
| 1 | 3.4277 | 4.4735 | 4.4726 |
| 2 | 3.2003 | 4.3774 | 4.437 |
| 3 | 3.3071 | 4.346 | 4.371 |
| 4 | 3.1377 | 3.6631 | 3.7228 |
| 5 | 2.8765 | 3.7038 | 3.7305 |
| 6 | 3.5104 | 3.5949 | 3.9923 |
| 7 | 2.7693 | 3.6806 | 3.7418 |
| 8 | 3.2637 | 3.8562 | 3.8778 |
| 9 | 2.7061 | 3.6717 | 3.7503 |
| 10 | 3.6519 | 4.4218 | 4.4433 |
| 11 | 2.775 | 3.8532 | 3.9071 |
| 12 | 2.5788 | 3.7084 | 4.3809 |
| 13 | 2.5115 | 3.6173 | 4.4123 |
| 14 | 2.5849 | 3.6757 | 3.744 |
| 15 | 2.8587 | 3.7795 | 3.8341 |
| 16 | 3.3078 | 3.7437 | 3.7525 |
| 17 | 3.2437 | 4.1746 | 4.1935 |
| 18 | 3.1636 | 3.9573 | 3.9857 |
| 19 | 2.7536 | 3.666 | 3.7333 |
| 20 | 2.7691 | 3.547 | 3.615 |
| Avg. SNR | 3.01987 | 3.875585 | 4.00489 |

Table 2: SNR values for STARE images generated by different pre processing methods

| Img | P | P+C | P+M+C |
|----------|---------|---------|---------|
| 1 | 1.4243 | 2.3542 | 2.59 |
| 2 | 1.4673 | 2.3579 | 2.5489 |
| 3 | 1.3666 | 2.3553 | 2.5846 |
| 4 | 1.3956 | 2.3363 | 2.5834 |
| 5 | 1.3282 | 2.3328 | 2.6 |
| 6 | 1.3932 | 2.3581 | 2.6055 |
| 7 | 1.3432 | 2.3887 | 2.6678 |
| 8 | 1.316 | 2.3894 | 2.7419 |
| 9 | 1.43 | 2.3883 | 2.6662 |
| 10 | 1.4473 | 2.4028 | 2.6893 |
| Avg. SNR | 1.39117 | 2.36638 | 2.62776 |

Table 3: SNR values for MRI images generated by different pre processing methods

| Img | P | P+C | P+M+C |
|----------|---------|----------|-----------|
| 1 | 1.8746 | 2.0749 | 2.121 |
| 2 | 1.5931 | 2.0108 | 2.0686 |
| 3 | 1.9389 | 2.1721 | 2.1605 |
| 4 | 1.8782 | 2.1426 | 2.1447 |
| 5 | 1.6902 | 2.0296 | 2.145 |
| 6 | 1.4474 | 1.9273 | 2.0343 |
| 7 | 1.1788 | 1.6269 | 1.7217 |
| 8 | 1.3016 | 1.7436 | 1.9029 |
| Avg. SNR | 1.61285 | 1.965975 | 2.0373375 |

Table 4: SNR values for Ultrasound images generated by different pre processing methods

| Img | P | P+C | P+M+C |
|----------|---------|-----------|---------|
| 01_h | 2.3549 | 2.8769 | 2.9667 |
| 02_h | 2.3677 | 2.8603 | 2.9409 |
| 03_h | 2.3337 | 2.8775 | 2.9791 |
| 04_h | 2.3054 | 2.7649 | 2.8093 |
| 05_h | 2.3466 | 2.7794 | 2.8441 |
| 01_dr | 2.3771 | 2.8213 | 2.9021 |
| 02_dr | 2.3708 | 2.7507 | 2.8099 |
| 03_dr | 2.3678 | 2.7402 | 2.8108 |
| 04_dr | 2.3543 | 2.7548 | 2.8247 |
| 05_dr | 2.3409 | 2.8162 | 2.8761 |
| 01_g | 2.4062 | 2.972 | 3.332 |
| 02_g | 2.3766 | 2.7994 | 2.8945 |
| 03_g | 2.3758 | 2.8716 | 2.9466 |
| 04_g | 2.3742 | 2.8665 | 2.9363 |
| 05_g | 2.3612 | 2.8441 | 2.9257 |
| Avg. SNR | 2.36088 | 2.8263867 | 2.91992 |

Table 5: SNR values for HRF images generated by different pre processing methods

| Img | R | P | P+C | P+M+C |
|-----------|--------|--------|--------|--------|
| 1 | 0.8905 | 0.9108 | 0.9563 | 0.9581 |
| 2 | 0.9159 | 0.8976 | 0.9553 | 0.955 |
| 3 | 0.8929 | 0.9003 | 0.9397 | 0.9408 |
| 4 | 0.8947 | 0.908 | 0.9586 | 0.9572 |
| 5 | 0.9296 | 0.9063 | 0.9578 | 0.9583 |
| 6 | 0.9165 | 0.9027 | 0.9506 | 0.951 |
| 7 | 0.8709 | 0.9086 | 0.9521 | 0.9529 |
| 8 | 0.9127 | 0.914 | 0.9476 | 0.9494 |
| 9 | 0.9158 | 0.919 | 0.957 | 0.9537 |
| 10 | 0.9219 | 0.9177 | 0.9571 | 0.9593 |
| 11 | 0.8752 | 0.9105 | 0.952 | 0.955 |
| 12 | 0.8772 | 0.9137 | 0.9535 | 0.9534 |
| 13 | 0.8917 | 0.9022 | 0.9504 | 0.9481 |
| 14 | 0.864 | 0.9192 | 0.9569 | 0.9581 |
| 15 | 0.8685 | 0.9362 | 0.9519 | 0.9577 |
| 16 | 0.8918 | 0.9097 | 0.9576 | 0.9574 |
| 17 | 0.8763 | 0.9156 | 0.9551 | 0.9535 |
| 18 | 0.8925 | 0.9208 | 0.9631 | 0.9643 |
| 19 | 0.9141 | 0.917 | 0.9598 | 0.9669 |
| 20 | 0.9262 | 0.9265 | 0.9658 | 0.9653 |
| Avg. Acc. | 0.8969 | 0.9128 | 0.9549 | 0.9558 |

Table 6: Accuracy of the proposed method for DRIVE images using different pre processing methods

3.2 Fréchet PDF based Matched Filter

Fréchet PDF is also known as extreme value PDF. It has skewed properties. It is described as follows:

$$f(x, y) = \alpha/\beta((d - \gamma)/\beta) \wedge (-(1 + \alpha))e \wedge (-(d - \gamma)/\beta) \wedge (-\alpha) \forall |y| \leq L/2 \quad (4)$$

where $\alpha > 0$, $\beta > 0$, $f(x, y)$: Fréchet-kernel, (x, y) : element's coordinate in the kernel, d : smallest distance between center and coordinate (x, y) , L : vessel's segment, α : shape parameter, β : scale parameter, γ : location parameter. Initially, vessels are assumed to be oriented across y axis and Fréchet kernel at origin. In Fréchet match-filter, shape parameter (α) manages the shapes of Fréchet curve. We can match Fréchet kernel as near as shape of vessel-profile by choosing suitable value of α . Accuracy of a match filter method depends on how closely that match filter kernel matches with shape of vessel profile. Other match filter methods such as Cauchy, Gumbel and Gaussian etc. have no such parameter. Therefore, Fréchet match filter generates better accuracy than other match filter methods by choosing suitable value of α .

In the literature, large scale experiments are carried out on various retinal data sets to select appropriate values of α , β , γ , L and n for the Fréchet filter and MSMO operator. The method is practically tested for distinct values of α (varying from 1 to 5 having interval 1), β (varying from 0 to 5.0 having interval 0.5), γ (varying from 0 to 5.0 having interval 0.5), L (varying from 1 to 15 having interval 1) and n (varying from 1 to 15 having interval 1) for every image of DRIVE data set. We found average accuracy 95.74 % for $\alpha=1$, $\beta=1$, $\gamma=1.72$, $L=11$ and $n=5$. The method is practically tested for distinct values of α (varying from 1 to 5 having interval 1), β (varying from 0 to 5.0 having interval 0.5), γ (varying from 0 to 5.0 having interval 0.5), L (varying

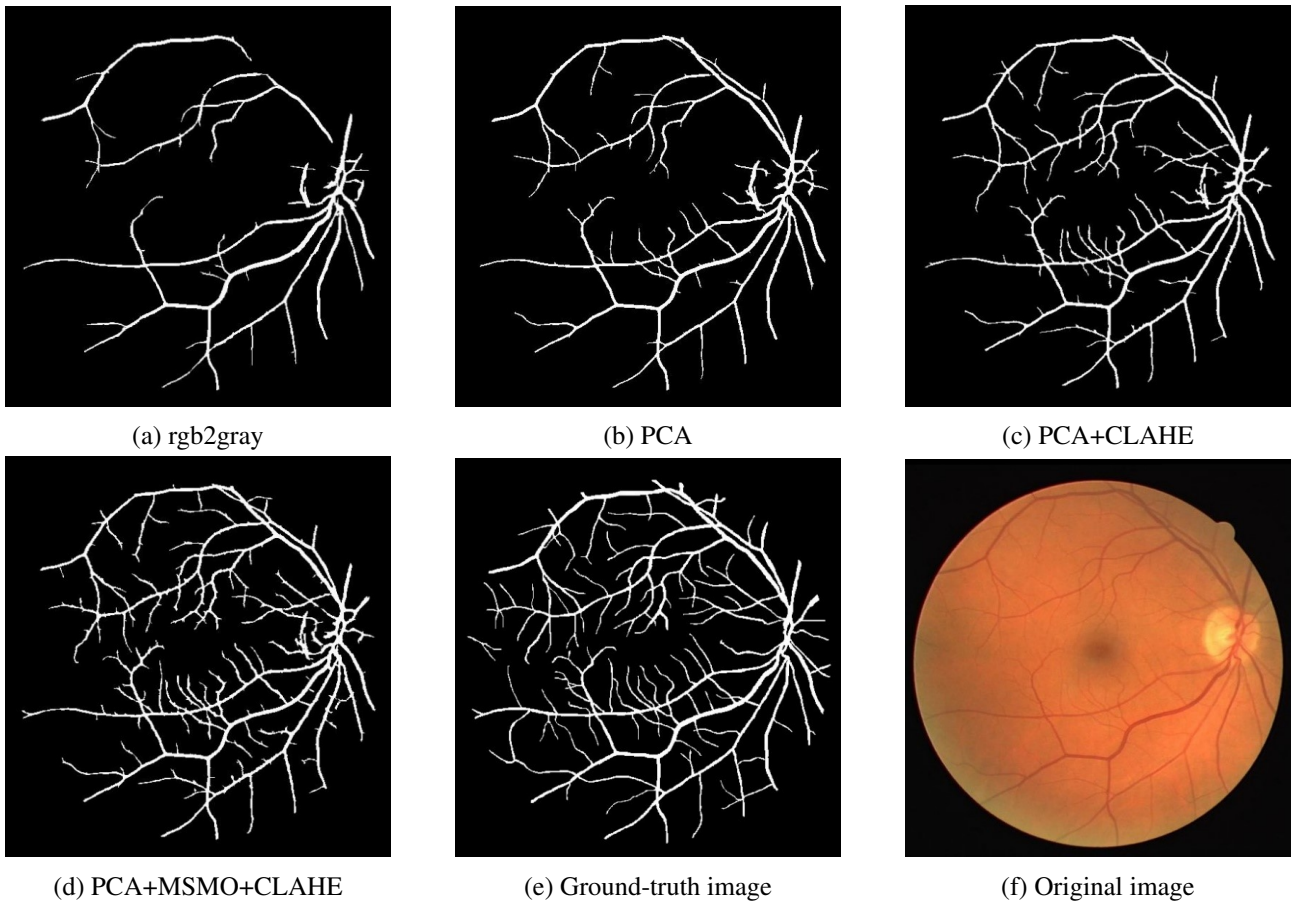


Figure 4: Segmented images of 16_test.tif using different pre processing methods for enhancement

from 1 to 15 having interval 1) and n (varying from 1 to 15 having interval 1) for every image of STARE data set. We found average accuracy 95.40 % for $\alpha=1$, $\beta=1.5$, $\gamma=4.5$, $L=11$ and $n=4$. The method is also practically tested for distinct values of α (varying from 1 to 5 having interval 1), β (varying from 0 to 5.0 having interval 0.5), γ (varying from 0 to 5.0 having interval 0.5), L (varying from 1 to 15 having interval 1) and n (varying from 1 to 15 having interval 1) for every image of HRF data set. We found average accuracy 94.02 % for $\alpha=1$, $\beta=1.81$, $\gamma=1.72$, $L=11$ and $n=10$.

To build Fréchet match filter kernel, optimal value of α , β , γ and L are chosen. At first, the kernel is assumed to be at origin. We assume a coordinate $C_x=[x, y]$ at the kernel. θ_i is the direction of i th kernel that compares to the vessel at angle θ . Retinal vessels are assumed to be scattered in every direction. Hence, it is required to convolve a kernel in each direction. When kernel and vessel are at exact angle, outcome is registered highest. To cover an entire image, 12- kernels with size 15×15 are implemented in each direction to detect retinal vessels by revolving kernel at 15 degree corresponding to preceding kernel. These Fréchet kernels having distinct directions are determined by equation (5).

$$R_{mi} = [\cos\theta \quad -\sin\theta; \sin\theta \quad \cos\theta] \quad (5)$$

The corresponding point $C_x=[R_x, R_y]$ in the revolved coordinate system is accomplished by equation (6).

$$P_{x_i} = [R_x, R_y] = P_x * R_{m_i}^T \quad (6)$$

After design of twelve Fréchet match filter kernels, these are convolved with a gray-scale image enhanced by the PCA+MSMO+CLAHE method to generate match-filter response (MFR) image. Figure 5 (c) shows a MFR image produced by the Fréchet match-filter.

3.3 Post-processing

Post processing operation has key role in the accuracy of vessel-segmentation. In the approach, we have applied appropriate and efficient methods for post processing operations. Post processing stage contains the vessels thresholding, length-filtering, masking and complimenting operations. These methods are described as follows:

3.3.1 Thresholding

In the post processing, first a thresholding algorithm is operated on the MFR image to generate binary image. For this, we apply the entropy-based optimal thresholding algorithm on MFR image produced by the Fréchet match-filter. This thresholding method is superior to other existing thresholding methods. The facts behind this are: in this method, possessions between pixel's intensities of MFR image manage the spatial structure in the threshold images and clearly identify vessels from their backgrounds. In this thresholding algorithm, first the gray level co-occurrence matrix (GLCM) is calculated. Thereafter GLCM is grouped in 4 quadrants. I as well as IV quadrants called 'local quadrants' contain gray-level transitions that happens within retinal vessels and backgrounds. Hence, these two quadrants are used to evaluate the local entropy for thresholding. To achieve this, probability of I as well as IV quadrants are determined. Then, entropies are calculated for these two quadrants. Highest value of summation of entropies is chosen as optimal-threshold value. Figure 5 (d) shows the extracted vessels from their background [32][33].

3.3.2 Length Filtering

Vessels extracted by thresholding methods have generally isolated and misclassified pixels. To remove these, we have applied the length filtering technique on extracted vessels. In the literature, we have applied the pixel label propagation and 8-connected neighborhood length filtering methods. The methods are efficient and appropriate in the case of retinal vessels. Figure 5 (e) shows that isolated and misclassified pixels are removed.

3.3.3 Masking

There is a probability that some artifacts may lie on retinal image boundary. To remove these artifacts, we have applied the masking technique. Mask images for STARE and DRIVE are available online. We take masking of input image and perform the logical AND function with corresponding segmented image to eliminate the artifacts. Figure 5 (e) shows that outer artifacts are removed.

3.3.4 Complimenting

In the last of post processing, we perform complement operation on the output image of masking operation. The obtained complemented image is then compared with the corresponding ground truth image for performance evaluation. Figure 5 (f) shows the complemented image.

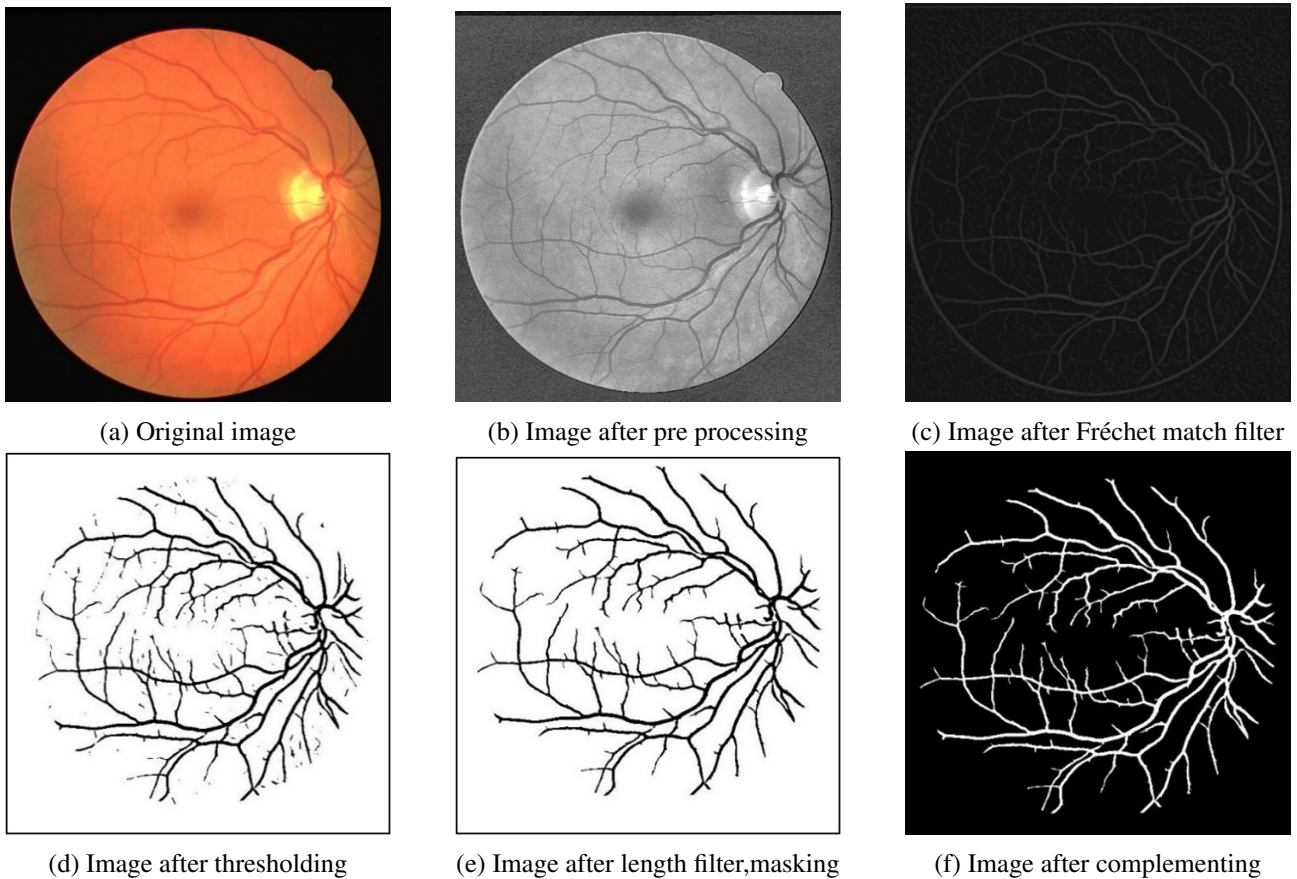


Figure 5: Different images generated at various phases of the proposed method for image 2_test.tif

4 Performance Evaluation

This section contains Experimental Setup, Performance Metrics and Result Analysis subsections.

4.1 Experimental Setup

The method is implemented and validated on various retinal image data sets: HRF, DRIVE and STARE which are standard and freely available online. These data sets briefly described as: DRIVE data set [34] contains 40 images. These images are classified in the training and test sets. Both the sets contain 20 images each. The test set contains two sets of manual segmented images while training contains single set of manually segmented images. Mask images for all 40 images are also available showing region of interest. Each image is captured using Canon CR-5 non mydriatic 3-CCD camera at 45° field of view (FOV). These acquired images contain 8 bits per plane. These images are cropped around FOV. Each image has 565 x 584 dimensions. STARE data set [30] has 400 raw images in which 20 images are selected at random. Out of them, 10 images are healthy and rest images are unhealthy. All the images are acquired by TOPCON-TRV-50 camera with 35° FOV. These images contain 8 bits/plane. Each image has 700 x 605 dimension. Ground truth images are provided by A. Hoover and V. Kouznetsova. HRF data set [4] has 45 original color images out of them 15 images are healthy, 15 images have glaucoma and 15 images have diabetic. HRF data-set has high resolution images. Images are acquired with Canon CR-1 fundus camera at 45° FOV. Every image of this data-set has dimension 3504 x 2336. The method is implemented as well as tested on the MATLAB R2017a on HP desktop having configuration 4-GB RAM, Intel (R), Xeon (R), E3, 3.4 GHz processor and 64 bits windows operating system.

4.2 Performance Metrics

Output of segmentation procedure is basically categorization of the pixels. Each pixel is either non vessel or vessel. Based on this, there may have four possible outputs. These are described as follows:

- True positive (TP): A pixel is identified as a vessel pixel in both segmented and ground truth image.
- True negative (TN): A pixel is identified as a non-vessel pixel in both segmented and ground truth.
- False negative (FN): A pixel is identified as non-vessel pixel in segmented but vessel in ground truth.
- False positive (FP): A pixel is identified as a vessel pixel in segmented and non-vessel in ground truth.

Quantitative performance metrics are calculated on these four outputs. The metrics used in the paper are described as follows:

- Specificity (SP): It is potential of an approach to identify non-vessel pixels. It is defined by equation (7).

$$SP = TN / ((TN + FP)) \quad (7)$$

- Sensitivity (SN): It is potential of an approach to identify vessel's pixels. It is described by equation (8).

$$SN = TP / ((TP + FN)) \quad (8)$$

- Accuracy (AC): It is ratio of total accurately categorized pixels of an image. It is defined by equation (9).

$$AC = ((TP + TN)) / ((TP + TN + FN + FP)) \quad (9)$$

- Precision (PR): It is a fraction of the pixels which are true-positive in all pixels identified as vessel pixels.

$$PR = TP / ((TP + FP)) \quad (10)$$

- F1 score: It is a weighted-average of sensitivity and precision. It is described by equation (11).

$$F1score = (2 * TP) / ((2 * TP + FP + FN)) \quad (11)$$

4.3 Result Analysis

4.3.1 Quantitative Analysis

In the literature, various quantitative measures like sensitivity, accuracy and specificity are computed. Values of these measures of the proposed method for each image of HRF, DRIVE and STARE data sets are reported in the Tables 7-9 respectively. Values of average specificity, sensitivity and accuracy of the proposed method are compared with various prominent methods of this field for DRIVE and STARE in the Table 10 and 11, respectively. Average accuracy of proposed method is 95.74 % which is found better by 0.30 %, 0.38 %, 0.52 %, 0.61 %, 0.64 %, 1.02 %, 1.05 %, 1.22 %, 3.05 %, 3.74 % and 4.78 % corresponding to currently available approaches viz; Saroj et al., GeethaRamani et al., Singh et al, Singh et al., Memari et al., Lam et al., Zhou et al., Martin et al., Zolfagharnasab et al., Amin et al. and AL-Rawi et al. respectively for DRIVE data set. Like the same way, average accuracy of proposed method is 95.40 % which is found better by 0.31 %, 0.40 %, 0.84 %, 0.91 %, 1.99 %, 2.70 %, 2.73 %, 2.80 %, 4.53 %, 5.64 % and 6.09 % corresponding to currently available approaches viz; Saroj et al., Sreejini et al., Kaba et al., Chen et al., Odstrcilik et al., Singh et al., Hoover et al., Palomera et al., Zhang et al., Kande et al. and Singh et al. respectively for STARE data set. Average accuracy of proposed method is 94.02 % for HRF data set. We have also calculated the precision and F1 score for evaluation of performance. Figure 6 shows the comparison statistics of precision and F1 score. Figure 7 shows the graphical representation of overall performance of the proposed method.

| Image | SP | AC | Image | SP | AC | Image | SP | AC |
|-------|--------|--------|-------|--------|----------|--------|----------|--------|
| 1 | 0.9944 | 0.9361 | 16 | 0.9872 | 0.9538 | 31 | 0.9918 | 0.9443 |
| 2 | 0.9934 | 0.941 | 17 | 0.9872 | 0.9429 | 32 | 0.9877 | 0.9404 |
| 3 | 0.994 | 0.927 | 18 | 0.9859 | 0.9434 | 33 | 0.9984 | 0.9489 |
| 4 | 0.9883 | 0.9409 | 19 | 0.9848 | 0.946 | 34 | 0.9962 | 0.9463 |
| 5 | 0.9975 | 0.9347 | 20 | 0.9829 | 0.949 | 35 | 0.9939 | 0.9464 |
| 6 | 0.9884 | 0.9408 | 21 | 0.9845 | 0.9358 | 36 | 0.993 | 0.9423 |
| 7 | 0.9981 | 0.9324 | 22 | 0.9845 | 0.935 | 37 | 0.993 | 0.9532 |
| 8 | 0.9936 | 0.9375 | 23 | 0.9828 | 0.9321 | 38 | 0.9868 | 0.9481 |
| 9 | 0.9987 | 0.9432 | 24 | 0.9914 | 0.9405 | 39 | 0.9908 | 0.9483 |
| 10 | 0.9933 | 0.9433 | 25 | 0.9829 | 0.9298 | 40 | 0.9938 | 0.9518 |
| 11 | 0.9888 | 0.9385 | 26 | 0.988 | 0.9343 | 41 | 0.992 | 0.9428 |
| 12 | 0.9902 | 0.9344 | 27 | 0.9729 | 0.9342 | 42 | 0.9847 | 0.9394 |
| 13 | 0.99 | 0.9352 | 28 | 0.9865 | 0.9413 | 43 | 0.9933 | 0.9464 |
| 14 | 0.9867 | 0.9339 | 29 | 0.975 | 0.9245 | 44 | 0.9903 | 0.9434 |
| 15 | 0.99 | 0.9421 | 30 | 0.9583 | 0.9224 | 45 | 0.9941 | 0.9404 |
| | | | | | Avg spe. | 0.9889 | Avg acc. | 0.9402 |

Table 7: Performance of the method for HRF dataset

| Image | SP | SN | AC |
|-------------|--------|--------|--------|
| 01_test.tif | 0.9722 | 0.8202 | 0.9548 |
| 02_test.tif | 0.9833 | 0.7645 | 0.9571 |
| 03_test.tif | 0.9591 | 0.7586 | 0.9553 |
| 04_test.tif | 0.9929 | 0.6592 | 0.9584 |
| 05_test.tif | 0.9889 | 0.7139 | 0.9585 |
| 06_test.tif | 0.9887 | 0.6453 | 0.9555 |
| 07_test.tif | 0.9784 | 0.7202 | 0.951 |
| 08_test.tif | 0.9797 | 0.6808 | 0.9596 |
| 09_test.tif | 0.9899 | 0.6653 | 0.9598 |
| 10_test.tif | 0.9847 | 0.6876 | 0.9563 |
| 11_test.tif | 0.9795 | 0.7137 | 0.9519 |
| 12_test.tif | 0.9823 | 0.7237 | 0.9561 |
| 13_test.tif | 0.9826 | 0.6918 | 0.9599 |
| 14_test.tif | 0.9696 | 0.7986 | 0.9581 |
| 15_test.tif | 0.9639 | 0.8112 | 0.9591 |
| 16_test.tif | 0.9826 | 0.7663 | 0.9589 |
| 17_test.tif | 0.9893 | 0.6322 | 0.9553 |
| 18_test.tif | 0.9825 | 0.7658 | 0.9614 |
| 19_test.tif | 0.9685 | 0.8608 | 0.9557 |
| 20_test.tif | 0.9847 | 0.7757 | 0.9655 |
| Average | 0.9802 | 0.7328 | 0.9574 |

Table 8: Performance of the method for DRIVE

| Image | SP | SN | AC |
|------------|--------|--------|--------|
| im0001.ppm | 0.962 | 0.8374 | 0.9501 |
| im0002.ppm | 0.9802 | 0.7566 | 0.9565 |
| im0003.ppm | 0.9528 | 0.7662 | 0.9334 |
| im0004.ppm | 0.9849 | 0.7254 | 0.9596 |
| im0005.ppm | 0.9825 | 0.7329 | 0.9583 |
| im0044.ppm | 0.9889 | 0.6216 | 0.9524 |
| im0077.ppm | 0.9725 | 0.7191 | 0.9485 |
| im0081.ppm | 0.9794 | 0.6516 | 0.9513 |
| im0082.ppm | 0.9925 | 0.6123 | 0.9609 |
| im0139.ppm | 0.9739 | 0.7485 | 0.9538 |
| im0162.ppm | 0.9766 | 0.7001 | 0.9522 |
| im0163.ppm | 0.987 | 0.6778 | 0.9558 |
| im0235.ppm | 0.9736 | 0.7085 | 0.9469 |
| im0236.ppm | 0.9644 | 0.7907 | 0.9495 |
| im0239.ppm | 0.9637 | 0.7961 | 0.9509 |
| im0240.ppm | 0.9846 | 0.7233 | 0.9602 |
| im0255.ppm | 0.9849 | 0.6519 | 0.956 |
| im0291.ppm | 0.9763 | 0.7786 | 0.9598 |
| im0319.ppm | 0.9697 | 0.832 | 0.9575 |
| im0324.ppm | 0.9834 | 0.7657 | 0.9665 |
| Average | 0.9766 | 0.7298 | 0.954 |

Table 9: Performance of the method for STARE

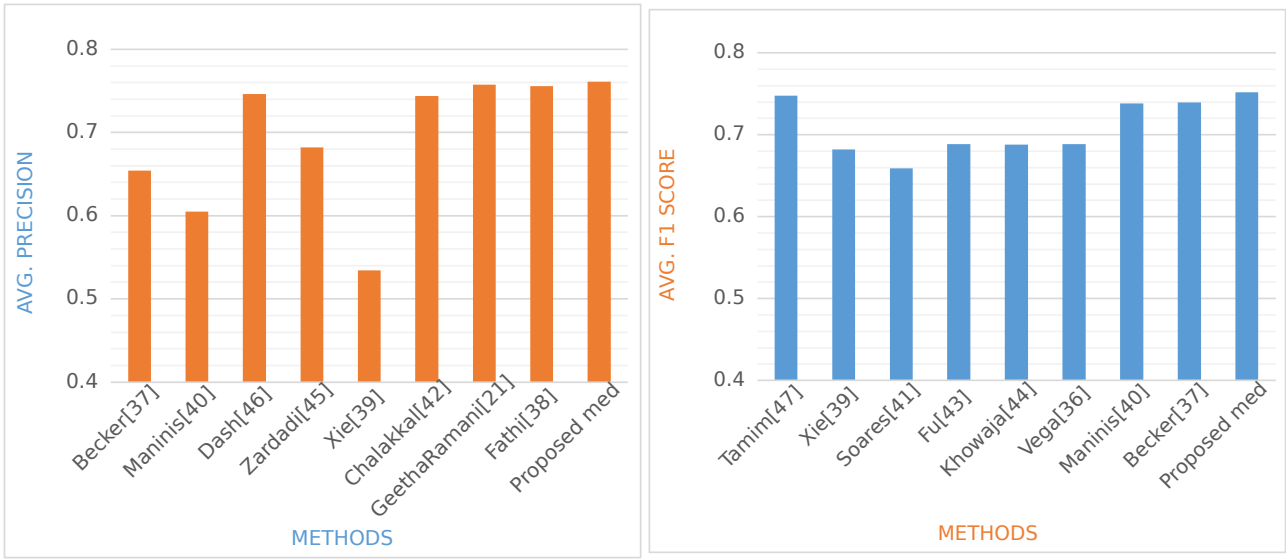


Figure 6: Comparison statistics of precision and F1 score for DRIVE dataset

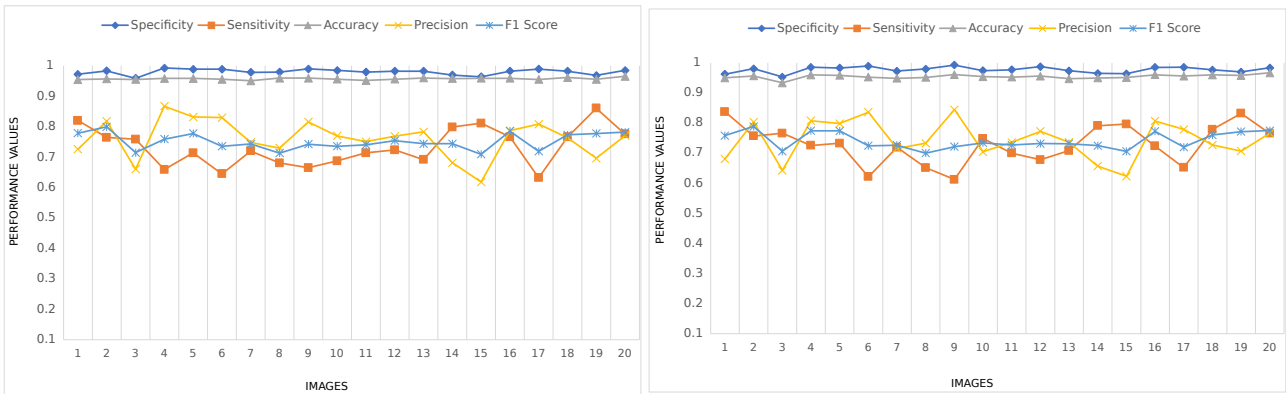


Figure 7: Graphical representation of the performance of the method for DRIVE and STARE, respectively

| Author, Year, Ref. | SP | SN | AC |
|---------------------|--------|--------|--------|
| Rawi et al. [28] | 0.9553 | 0.5993 | 0.9096 |
| Zolfa. et al. [9] | 0.9714 | 0.6239 | 0.9269 |
| Memari et al. [1] | 0.965 | 0.782 | 0.951 |
| Zhou et al. [5] | 0.9674 | 0.8078 | 0.9469 |
| Geetha. et al. [21] | 0.9778 | 0.7079 | 0.9536 |
| Amin et al. [19] | | | 0.92 |
| Marín et al. [20] | 0.9801 | 0.7067 | 0.9452 |
| Singh et al. [26] | 0.9739 | 0.7171 | 0.9513 |
| Lam et al. [15] | | | 0.9472 |
| Singh et al. [7] | 0.9708 | 0.7594 | 0.9522 |
| Saroj et al. [6] | 0.9761 | 0.7307 | 0.9544 |
| Proposed approach | 0.9802 | 0.7328 | 0.9574 |

Table 10: Comparison of methods for DRIVE

| Author, Year, Ref. | SP | SN | AC |
|--------------------|--------|--------|--------|
| Hoover et al. [30] | 0.9567 | 0.6751 | 0.9267 |
| Kande et al. [27] | | | 0.8976 |
| Zhang et al. [31] | 0.9736 | 0.7373 | 0.9087 |
| Pal. et al. [35] | 0.9449 | 0.769 | 0.926 |
| Ods. et al. [4] | 0.9512 | 0.7847 | 0.9341 |
| Kaba et al. [8] | 0.9672 | 0.7619 | 0.9456 |
| Sre. et al. [13] | 0.9687 | 0.7172 | 0.95 |
| Singh et al.[11] | 0.9072 | 0.8389 | 0.8931 |
| Chen et al. [14] | 0.9696 | 0.7295 | 0.9449 |
| Singh et al. [7] | 0.9376 | 0.7939 | 0.927 |
| Saroj et al. [6] | 0.9724 | 0.7278 | 0.9509 |
| Proposed approach | 0.9766 | 0.7298 | 0.954 |

Table 11: Comparison of methods for STARE

4.3.2 Qualitative Analysis

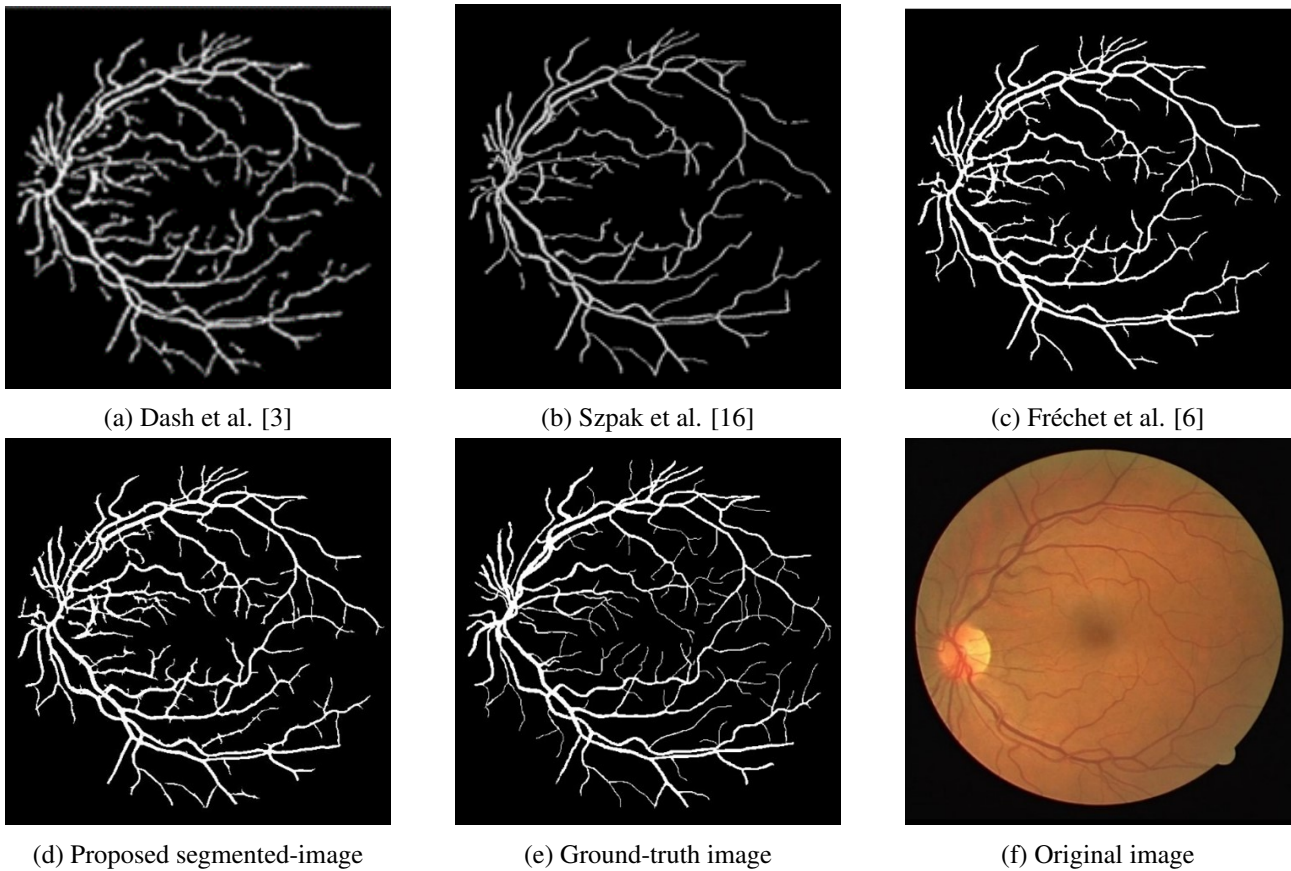


Figure 8: Comparison of segmented images generated by various methods for image 1_test.tif

5 Conclusion

Diabetes, hypertension, obesity, glaucoma etc. have become greater threat to human life today. Information about these diseases can be obtained from retinal blood vessels. Therefore, many researchers have proposed various methods to segment retinal blood vessels. Most researchers have focused only on segmentation process. Pre processing plays vital role in accuracy of vessel segmentation. Hence, the proposed method not only apply efficient segmentation method i.e. the Fréchet match filter but also introduces a novel method called MSMO method for pre processing. We have tested and validated the proposed method on DRIVE, STARE and HRF data sets. The average accuracy of the proposed method is 95.74 %, 95.40 % and 94.02 % for DRIVE, STARE and HRF data sets, respectively. Obtained results demonstrate that the proposed method outperforms over latest and prominent findings of this area. The cause of improved performance is the better pre processing and segmentation methods.

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